**CSCI 570 Spring 2016 Discussion 4**

1. Hardy decides to start running to work in San Francisco city to get in shape. He prefers a route that goes entirely uphill and then entirely downhill so that he could work up a sweat uphill and get a nice, cool breeze at the end of his run as he runs faster downhill. He starts running from his home and ends at his workplace. To guide his run, he prints out a detailed map of the roads between home and work with k intersections and m road segments (any existing road between two intersections). The length of each road segment and the elevations at every intersection are given. Assuming that every road segment is either fully uphill or fully downhill, give an efficient algorithm to find the shortest path (route) that meets Hardy's specifications. If no such path meets Hardy's specifications, your algorithm should determine this. Justify your answer.

2. Suppose you have two min-heaps, A and B, with a total of *n* elements between them. You want to discover if A and B have a key in common. Give a solution to this problem that takes time O(n log n) and explain why it is correct. Give a brief explanation for why your algorithm has the required running time.

For this problem, do not use the fact that heaps are implemented as arrays; treat them as abstract data types.

3. Let *H* be a min-heap storing *n* keys. Give an efficient algorithm to report all keys in *H* that are less than or equal to a given value *x* (which may or may not be in the heap). Ideally, your algorithm should run in time *O(k)*, where *k* is the number of keys reported. Note that this is independent of *n*.

For example, if your heap is {4, 5, 6, 15, 9, 7, 20, 16, 25, 14, 12, 11, 8} and *x*=7, your algorithm should output 4, 5, 6 and 7 (not necessarily in that order).